CONVECTIVE HEAT AND MASS EXCHANGE IN ASYMMETRIC FLUIDS

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Equations and boundary conditions of the theory of highly intensive convective heat- and mass-exchange processes in nonlinear asymmetric fluids are formulated. The specifics of the transport process in macrocapillaries and a slightly rarefied gas are taken into account. Problems of free convection in a vertical channel, diffusion of Brownian particles, and heat exchange in a tube, are solved.

Theoretical investigation of convective heat- and mass-exchange processes is constructed on the basis of specific models of continuous media and the transport processes therein. As science develops, and in connection with the demands of practice, the models of continuous media and the transport processes therein become complicated [1-11]. From our viewpoint, the following generalizations of classical models of a continuous medium are of great interest: 1) the construction of a nonlinear mechanics of continuous media; and 2) the creation of an asymmetric mechanics of solids and fluids [6-30].

Transport processes in fluid and solid media whose rheological behavior is described by linear and nonlinear asymmetric mechanics have practically not been examined up to now.

An attempt is made herein to formulate the fundamental equations and boundary conditions of a theory of highly intensive transport processes in asymmetric fluids. The specifics of these processes is illustrated in a number of specific linear heat- and mass-exchange processes. It is shown that the need to take account of the asymmetric properties of the medium arises naturally in a study of heat and mass exchange in disperse media and rheological systems.

Nonlinear Asymmetric Mechanics of Continuous Media

In constructing the ordinary continual mechanics of continuous media it is assumed that the stress state is determined entirely by a symmetric stress tensor. This corresponds to the ideal model of a continuous media when physically an infinitesimal volume can be considered spherically symmetric and the interaction between such volumes can be computed by using central forces. If such a representation is impossible under some considerations (and such considerations appear more and more often), the effect of one volume element of a medium on another must be described by using noncentral forces and moments which can be connected in the usual way with the stress tensor σ_{ik} and the micromoment tensor μ_{ik}. It hence turns out that the stress may be asymmetric, and its symmetry in the customary theory is associated with neglecting not only the rotational interactions between volume elements of the medium, but also the external spatially distributed effect on the medium.

In principle, taking account of the rotational interaction between volume elements is apparently important in studying the convective heat and mass exchange in disperse systems of non-Newtonian fluids. A theoretical study of transport processes in such systems is often carried out within the scope of a single-fluid approximation, i.e., within the scope of the representation of a continuous medium by using the reduced MacAdam parameters [4], say. However, particles of the dispersed phase, or individual macromolecule elements possess intrinsic rotation which affects their translational motion; and this can be taken into account within the scope of the asymmetric mechanics of a continuous medium.

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Within the scope of asymmetric mechanics, the strain state is described by the strain tensor e_{ik}, the strain rates $\dot{e}_{ik}$, the strain moment tensor r_{ik}, and the strain moment rates $\dot{r}_{ik}$, while the dynamics of a moving fluid is described by the velocity vector v_i and the intrinsic angular velocity vector ω_i.

As usual, the thermodynamic state is described by the temperature T, the density ρ and the pressure p, which are interrelated by the equation of state

$$F(p,\ \rho,\ T) = 0. \tag{1}$$

Let us derive the fundamental equations of motion. Let us limit ourselves to the analysis of fluid media, where the strain state is described only by the strain rate tensor $\dot{e}_{ik}$ and the strain moment rate tensor $\dot{r}_{ik}$. For finite strains the strain rates $\dot{e}_{ik}$ and $\dot{r}_{ik}$ are related to the field of the velocities v_i and the intrinsic angular velocities ω_i as follows:

$$\dot{e}_{ik} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \frac{1}{2}\,\delta_{ij}\frac{\partial v_k}{\partial x_j}\frac{\partial v_k}{\partial x_j} + \frac{1}{2}\,(1-\delta_{ij})\frac{\partial v_k}{\partial x_i}\frac{\partial v_k}{\partial x_j} - \varepsilon_{ijk}\omega_j.$$

$$\dot{r}_{ik} = \frac{\partial \omega_i}{\partial x_k}, \tag{2}$$

where ε_{ijk} is the Levi-Civitas tensor.

Let us write the rheological equations of state as

$$\sigma_{ik} = -p\delta_{ik} + \eta_V\dot{e}_{jj}\delta_{ik} - (\eta - \gamma)\,\dot{e}_{ik} + (\eta + \gamma)\,\dot{e}_{ki} + \eta_1\dot{e}_{ij}\dot{e}_{jk} + \eta_2\dot{e}_{kj}\dot{e}_{ji}, \tag{3}$$

$$\mu_{ik} = \eta_3\dot{r}_{jj}\delta_{ik} + \eta_4\dot{r}_{ik} + \eta_5\dot{r}_{ki} + \eta_6\dot{r}_{ij}\dot{r}_{jk} + \eta_7\dot{r}_{kj}\dot{r}_{ji} \qquad \eta,\ \eta_V,\ \eta_i,\ \gamma \geqslant 0. \tag{4}$$

However, let us consider the coefficients of volume η_V, shear η, and rotational η_i viscosity, as well as a coefficient characterizing the measure of "coupling" between the translational and intrinsic rotational motions of the fluid particle γ to be functions i_n and I_n of the fundamental tensor invariants $\dot{e}_{ik}$, $\dot{r}_{ik}$, or $(\sigma_{ik},\ \mu_{ik})$;

$$i_1 = \dot{e}_{nn},\ I_1 = \dot{r}_{nn},$$

$$i_2 = \dot{e}_{nn}\dot{e}_{kk} - \dot{e}_{nk}\dot{e}_{kn},\ I_2 = \dot{r}_{nn}\dot{r}_{kk} - \dot{r}_{nk}\dot{r}_{kn}, \tag{5}$$

$$i_3\,\mathrm{Det}\,|\dot{e}_{ik}|,\quad I_3 = \mathrm{Det}\,|\dot{r}_{ik}|.$$

In the general case, the arbitrary, but sufficiently smooth, dependence of the physical parameters of the medium Y_i (we understand Y_i to be the coefficient of viscosity, γ, etc.) on the tensor invariants can be represented by power series

$$Y_i = \sum_{n_i} \alpha_{n_i}\, i_1^{n_1}\, i_2^{n_2}\, i_3^{n_3}\, I_1^{n_4}\, I_2^{n_5}\, I_3^{n_6}. \tag{6}$$

Their effective values [4] should be understood for the coefficients of viscosity for a single-fluid description of disperse systems of the liquid (gas) – solid particle suspension type.

The laws of conservation of momentum and the moment of momentum are in the general case

$$\frac{\partial \sigma_{ik}}{\partial x_k} + \rho f_i = \rho\,\frac{dv_i}{dt}, \tag{7}$$

$$\frac{\partial \mu_{ik}}{\partial x_k} + \sigma_{nm}\varepsilon_{imn} + \rho m_i = \rho\frac{dM_i}{dt}, \tag{8}$$

where ρ is the density of an asymmetric fluid; m_i is the density of the spatially distributed forces and moments; d/dt is the substantial derivative.

It is necessary to add the continuity equation

$$\frac{\partial \rho}{\partial t} + \mathrm{div}\,(\rho v) = 0. \tag{9}$$

to (7) and (8).

Intensive Heat and Mass Exchange in Nonlinear Asymmetric Fluids

The energy and mass conservation laws in moving asymmetric fluids in which heat conduction, diffusion, and cross effects occur and dissipative processes for a binary mixture (C, $\rho - C$) are taken into account, are

$$\frac{\partial E}{\partial t} = \operatorname{div}(\lambda \nabla T + D_2 \nabla C) + \sigma_{ik}\dot{e}_{ik} + \mu_{ik}\dot{r}_{ik}, \tag{10}$$

$$\frac{\partial C}{\partial t} = \operatorname{div}(D \nabla C + D_1 \nabla T), \tag{11}$$

where λ and D are the heat conduction and diffusion coefficients; D_2 is the effective DuFour coefficient; and D_1 is the effective coefficient of thermal diffusion.

The last member in (10), associated with the work of the volume and surface moments, has been introduced in [14].

For intensive heat- and mass-exchange processes the coefficients λ, D, D_1, D_2 must be considered functions of the temperature T, the concentration C, the tensor invariants $\dot{e}_{ik}$, $\dot{r}_{ik}$, and also the square of the temperature and concentration gradients, $(\nabla T)^2$ and $(\nabla C)^2$. The coefficient γ and the viscosity coefficients hence become functions of the temperature, concentration, $(\nabla T)^2$ and $(\nabla C)^2$

$$Y_i = Y_i\left[i_n, I_n, C, T, (\nabla T)^2, (\nabla C)^2\right]. \tag{12}$$

The formulated system (1)-(12) permits solution of the problem of highly intensive convective heat- and mass-exchange processes in asymmetric fluids. The nature of the forces and moments in (7) and (8) can be distinct. In particular, these forces can be gravitational, Coriolis, electromagnetic, etc.

Boundary Conditions

The dynamics of an asymmetric fluid is described by two kinds of velocities, the translational v_i and the intrinsic rotation ω_i. Hence, more complex boundary conditions must be assigned than for the ordinary fluid.

Firstly, the translational velocity on the solid boundary is determined by the conditions of partial adhesion and slip, which we shall consider to be proportional to the derivative of the velocity with respect to the normal to the surface in a first approximation, as well as by the derivative of the temperature and concentration along the tangent to the boundary surface

$$v_s - v_g = \alpha_1 \frac{\partial v}{\partial n} + \alpha_2 \frac{\partial T}{\partial s} + \alpha_3 \frac{\partial C}{\partial s}, \tag{13}$$

where v_g is the velocity of the boundary; and v_s is the slip velocity.

The boundary conditions, derived in [14], for the velocity components ω_i are

$$\alpha_{ik}\left(\omega_k - \frac{1}{2}\operatorname{rot} v_k\right) = \eta_1 \operatorname{div}\vec{\omega}\delta_{ik} + \eta_4 \frac{\partial \omega_i}{\partial x_k} + \eta_5 \frac{\partial \omega_k}{\partial x_i}, \tag{14}$$

where α_{ik} are the coefficients of rotational surface friction. The mechanism of asymmetric fluid interaction with solid surfaces has still not been clarified at present, hence, tests should show how well the boundary conditions have been formulated.

The boundary conditions for the temperature and concentration are written also taking account of the jumps in temperature and concentration:

$$T_s - T_g = \varkappa_T \frac{\partial T}{\partial n} + \varkappa_v \frac{\partial v}{\partial s} + \varkappa_C \frac{\partial C}{\partial n}, \tag{15}$$

$$C_s - C_g = \varkappa_C \frac{\partial C}{\partial n} + \varkappa_T \frac{\partial T}{\partial n} + \varkappa_v \frac{\partial v}{\partial s}, \tag{16}$$

where $T_s - T_g$ is the temperature jump; $C_s - C_g$ is the concentration jump; and $\varkappa_C$ $\varkappa_C$, $\varkappa_T$, and κ_V are phenomenological coefficients.

Taking account of the influence of the jumps in temperature, concentration and velocity in the formulated boundary conditions (13)-(16) is particularly important in studying convective heat- and mass-exchange processes of asymmetric fluids in macro- and microcapillaries, as well as in rarefied asymmetric gases.

Let us illustrate the singularities of convective heat- and mass-exchange processes in asymmetric fluids by a number of problems.

Free Convection in a Vertical Channel

Let us examine the one-dimensional problem of free convection in a vertical channel. Let us consider that there exist only the velocity component in the gravitational field direction v_y, dependent on the x coordinate ($v_y = v(x)$) and the z component of the intrinsic angular velocity vector $\omega_z = \omega(x)$.

It is seen from the system (7)-(8) that the sole reason for the origin of a stationary rotational motion in the absence of volume distributed moments ($\mu_{ik} = 0$) is the inhomogeneous velocity field ($v = v(x)$ in a plane vertical channel). This field can excite the z velocity component of the intrinsic rotational motion.

Let us find the velocity fields $v(x)$ and $\omega(x)$ within a slot if the temperature of the solid slot walls is kept constant, but different, and the velocities v and ω thereon are zero. The formulated problem is described by the following system of dimensionless equations

$$\frac{d^2\omega}{dx^2} + A_1\left(\omega - \frac{dv}{dx}\right) = 0,$$
$$\frac{d^2v}{dx^2} - A_2\frac{d\omega}{dx} - \mathrm{Gr}\,T = 0,$$
$$\frac{d^2T}{dx^2} = 0 \qquad (17)$$

and the boundary conditions

$$v(\pm 1) = \omega(\pm 1) = 0,$$
$$T(1) = -1, \quad T(-1) = 1, \qquad (18)$$

where

$$x = \frac{x'}{a}, \quad T = \frac{T - T_0}{\theta}, \quad v = \frac{\mathrm{v}}{a},$$
$$\beta = -\frac{1}{\rho}\frac{\partial\rho}{\partial T}g, \quad \mathrm{Gr} = \frac{\beta\theta a^3}{\nu^2},$$
$$A_1 = \frac{\gamma a^2}{\eta_2}, \quad A_2 = \frac{2\gamma}{\eta + \gamma}.$$

Let us write down the solution of the problem

$$T = -x, \qquad (19)$$

$$\omega = C_2\left\{\begin{matrix}\mathrm{ch}\, sx\\ \cos sx\end{matrix}\right\} - \frac{A_1\mathrm{Gr}}{s^2}\frac{x^2}{2} + \frac{A_1\mathrm{Gr}}{s^4} + \frac{A_1C_1}{s^2}, \qquad (20)$$

$$v = C_1x - \frac{\mathrm{Gr}}{6}x^3 + A_2\left[\frac{C_2}{s}\left\{\begin{matrix}\mathrm{sh}\, sx\\ \sin sx\end{matrix}\right\} - \frac{A_1\mathrm{Gr}}{6s^2}x^3 + \frac{A_1(\mathrm{Gr} + C_1)}{s^2}x\right], \qquad (21)$$

where the constants C_i and s are:

$$s^2 = A_1 - A_1A_2,$$

$$C_1 = \frac{\mathrm{Gr}\left[\frac{1}{6} - A_1A_2\left(\frac{1}{2s^3} - \frac{1}{s^5}\right)\left\{\begin{matrix}\mathrm{th}\, s\\ \mathrm{tg}\, s\end{matrix}\right\} + \frac{A_1A_2}{s^2}\left(\frac{1}{6} - \frac{1}{s^2}\right)\right]}{\left[1 - \frac{A_1A_2}{s^3}\left\{\begin{matrix}\mathrm{th}\, s\\ \mathrm{tg}\, s\end{matrix}\right\} + \frac{A_1A_2}{s^2}\right]}, \quad C_2 = \frac{A_1\mathrm{Gr}}{s^2\cos s}\left(\frac{1}{2} - \frac{1}{s^2} - \frac{C_1}{\mathrm{Gr}}\right).$$

An indeterminacy appears in the solution (20), (21) in the case $s = 0$, $A_2 = 1$. Passing to the limit as $A_2 \to 1$, we obtain the solution of the problem as

$$v = \mathrm{Gr}\left\{\left[\frac{1}{6} - A_1\frac{5-A_1}{60(3-A_1)}\right](x-x^3) + \frac{A_1}{120}(x-x^5)\right\}, \quad (22)$$

$$\omega = A_1\,\mathrm{Gr}\left\{\frac{5-A_1}{30-10A_1}\,\frac{x^2}{2} - \frac{x^4}{24} + \frac{A_1-15}{120(30-A_1)}\right\}. \quad (23)$$

It is seen from the solution (20)-(23) that the flow symmetry remains the same as in the case of an ordinary fluid. In all cases the location of the velocity extrema shifts toward the slot walls, depends on the slot size, and for $s = 0$ is determined by the formula

$$x = \left\{\frac{-3D_1 \pm \sqrt{9D_1^2 + 20D_2(D_1+D_2)}}{10D_2}\right\}^{1/2},$$

$$D_1 = \frac{1}{6} - A_1\frac{5-A_1}{60(3-A_1)}, \qquad D_2 = \frac{A_1}{120}.$$

Approximate expressions can be presented for the magnitudes and extrema of the velocity in the case $A_1 \ll 1$:

$$v = \frac{\mathrm{Gr}}{6}\left[(x-x^3) + A_1A_2\left(\frac{x^5}{20} - \frac{x^3}{6} - \frac{7}{60}x\right)\right],$$

$$\omega = -\frac{\mathrm{Gr}}{6}A_1\left(\frac{x^4}{4} - \frac{x^2}{2} + \frac{1}{40}\right),$$

$$x_{\max} = \pm\frac{1}{\sqrt{3}} \pm A_1A_2\frac{23}{90(2\sqrt{3}-1)}.$$

If the coefficient of "cohesion" of the translational and rotational motion γ vanishes, the profile of the translational velocity becomes cubic (21), as in an ordinary fluid, and intrinsic rotation is not excited.

As is clear from the physical sense, the magnitude of the translational velocity in an asymmetric fluid is less, other conditions being equal, than in an ordinary fluid. Indeed, the energy delivered to the slot walls is now expended in dissipative processes associated with the rotational as well as the translational motion.

Diffusion of Suspended Particles in an Asymmetric Fluid

Let us determine the coefficient of diffusion of Brownian particles in an asymmetric fluid. It is known that the diffusion coefficient is found from the Einstein formula [11, 12]

$$D = KTb. \quad (24)$$

The mobility b is determined from the equation

$$\mathbf{v} = b\mathbf{F}, \quad (25)$$

where $\mathbf{v}$ is the velocity of the Brownian particle, and $\mathbf{F}$ is the motive force which is in the case of spherically symmetric particles [14]

$$F = 6\pi\eta R^*\mathbf{v}, \quad (26)$$

$$R^* = R\left[1 + \frac{k^2A^{-2}}{1-k^2A^{-2}+k(1+\delta_2 k)}\right]; \quad k = \frac{R}{k_2};$$

$$k_2^2 = -\frac{1}{2}\eta_5(\eta+\gamma)\eta^{-1}\gamma^{-1}; \quad \delta_2 = [2 + \eta_4\eta_5^{-1} + dR\eta_5^{-1}]^{-1}.$$

Substituting (26) into (25), we find b. The expression obtained for the mobility is then substituted into (24), yielding

$$D = \frac{KT}{6\pi\eta R}\left[1 + \frac{k^2A^{-2}}{1-k^2A^{-2}+k(1+\delta_2 k)}\right]^{-1}. \quad (27)$$

Let us present two limit cases of (27) when $\gamma/\eta \gg 1$, $\alpha = \infty$, and $\alpha = 0$:

$$\alpha = 0 \quad D = \frac{KT}{6\pi\eta R\,[1 + 2A^{-1}(A+2)^{-1}]},$$

$$\alpha \to \infty \quad D = \frac{KT}{6\pi\eta R\,(1 + A^{-1})}.$$

Solution of Certain Heat-Exchange Problems in an Asymmetric Fluid

The exact solution of the classical problem of heat exchange in the steady-state mode of single-phase fluid flow in a circular cylindrical tube with constant heat flux to the wall is successfully obtained.

The steady-state velocity profile in a circular cylindrical tube with constant pressure gradient has been obtained in [14]. Considering the heat-exchange process steady and taking into account axial symmetry, let us write down the energy equation in dimensionless form

$$\frac{\partial^2 T}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial T}{\partial \rho} = \frac{v_m B}{\chi}\left[1 - \rho^2 + \frac{2}{A^2}\,\frac{J_0(k\rho) - J_0(k)}{k^{-1}J_1(k) + \delta_1 J_2(k)}\right], \tag{28}$$

$$\rho = \frac{r}{R};\quad A = \sqrt{\frac{2\eta}{\eta_5}}\,R;\quad \delta_1 = 1 - \frac{\eta_4}{\eta_5} - \frac{\alpha R}{\eta_5};\quad v_m = -\frac{1}{4\eta}\frac{\partial p}{\partial z}R^2.$$

The physical fluid parameters are assumed constant.

Let us seek the temperature distribution assuring constant heat flux to the wall in the form

$$T = Bz + f(\rho). \tag{29}$$

We obtain the following equation for the function $f(\rho)$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{df}{d\rho}\right) = \frac{v_m B R^2}{\chi}\left[1 - \rho^2 + \frac{2}{A^2}\,\frac{J_0(k\rho) - J_0(k)}{k^{-1}J_1(k) + \delta J_2(k)}\right],$$

whose solution, bounded at zero, is

$$f = \frac{1}{4}\left[1 - \frac{2J_0(k)}{M}\right]\rho^2 - \frac{1}{16}\rho^4 - J_0(k\rho)\frac{2}{k^2 M} + \frac{4 + k^2}{2k^2 M}J_0(k) - \frac{3}{4},$$

$$M = A^2\,[k^{-1}J_1(k) + \delta J_2(k)]. \tag{30}$$

The solution obtained is easily integrated. The first three members yield the solution of the problem in the ordinary viscous, heat-conducting fluid, while the rest are associated just with the asymmetric properties.

Let us calculate the Nusselt criterion referred to the diameter 2R

$$\mathrm{Nu} = \frac{-2R\,(df/dr)_R}{f_{av}}. \tag{31}$$

Here f_{av}, the mean value of the temperature, over the cross section, is defined by the formula

$$f_{av} = \frac{4\pi\int_0^R f v r\,dr}{\pi R^2 v_m}.$$

For small values of $k\rho$ ($\theta \neq 0$, γ) simple approximate expressions can be given for the velocity, the function f and the Nusselt criterion. Limiting ourselves to terms of the order of $(k\rho)^2$ inclusive, in the Bessel function J_0, J_1, J_2, we obtain:

$$v = v_m(1 - \rho^2)(1 + N), \tag{32}$$

$$f = \frac{v_m B R^2}{4\chi}(1 + N)\left(\rho^2 - \frac{1}{4}\rho^4 - \frac{3}{4}\right), \tag{33}$$

$$\mathrm{Nu} = 4.36(1 - N), \tag{34}$$

$$N = 4\frac{\gamma}{\eta+\gamma}\left\{1 - \frac{\eta\gamma R^2}{4\eta_5(\eta+\gamma)}\left[2\frac{\eta_4+\alpha R}{\eta_5} - 1\right]\right\}. \tag{35}$$

It is interesting to note that the Nusselt number in an asymmetric fluid becomes a function of the tube radius.

It is seen from (35) that if

$$2(\eta_4 + \alpha R) < \eta_5, \tag{36}$$

then the Nusselt number is less than in the case of an ordinary fluid. But for a given asymmetric fluid, the inequality (36) is spoiled as the tube radius R grows, and the Nusselt number can become larger than in an ordinary fluid. Upon compliance with the condition

$$2(\eta_4 + \alpha R) = \eta_5$$

the Nusselt number in an ordinary and an asymmetric fluid agrees. Let us analyze the expression (30) for the temperature.

The temperature at a fixed point within the tube will also differ from the temperature in a symmetric fluid. Thus, upon compliance with condition (36), the temperature will be greater than in an ordinary fluid, and conversely.

Let us turn now to the classical Gretz–Nusselt problem. In a complete formulation such a problem in an asymmetric fluid is quite awkward. Hence, its solution is presented in approximate small $k\rho$. The dimensionless energy equation for the parabolic flow mode (32) is

$$\frac{\partial^2 T}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial T}{\partial \rho} = \frac{v_m R}{\chi}(1-\rho^2)(1+N)\frac{\partial T}{\partial z}.$$

Let us use the notation

$$a_1 = \frac{\chi}{v_m R(1+N)}.$$

Its solution under the boundary conditions

$$\rho = 1 \quad (z > 0), \quad T = 0,$$
$$z = 0 \quad (\rho < 1), \quad T = 1$$

is known:

$$T = \sum_{k=1}^{\infty} C_k \vartheta_k(\rho)\, e^{-\beta_k^2 a_1 z},$$

$$\vartheta_k(\rho, \beta_k) = 1 - \frac{\beta_k^2}{4}\rho^2 + \frac{3}{2\cdot 4!}\left(\beta_k^2 + \frac{\beta_k^4}{4}\right)\rho^4 + \ldots,$$

$$\beta_1 = 2.705, \quad \beta_2 = 6.66, \quad \beta_3 = 10.6,$$

where Nusselt has found the first three coefficients

$$C_1 = 1.477, \quad C_2 = -0.810, \quad C_3 = 0.385.$$

The distinction between the considered problem and the case of heat exchange in an ordinary fluid is that the dependence of the dimensionless temperature on the longitudinal z coordinate varies. Thus, for a fixed value of z the temperature of an asymmetric fluid can be greater or less than in an ordinary fluid, depending on condition (36).

Let us stress again that the predicted effects of the dependence of the Nusselt number on the tube radius, the position of the velocity extrema in free convection on the slot size, the deviation of the values of the diffusion coefficient from the classical value, the anomaly in the temperature distribution during heat exchange in tubes will apparently be most noticeable in convective heat- and mass-exchange processes in dispersions of solid particles and macromolecular compounds in low-molecular fluids, polyelectrolytes, physiological fluids, etc.

LITERATURE CITED

1. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).
2. A. V. Lykov, Theoretical Principles of Structural Thermophysics [in Russian], Minsk (1961).
3. A. V. Lykov and B. M. Berkovskii, in: Heat and Mass Exchange in Non-Newtonian Fluids [in Russian], Énergiya (1968).
4. A. V. Lykov, Izv. Akad. Nauk BSSR, Ser. Fiz.-Tekh. Nauk, No. 1 (1965).
5. L. I. Sedov, Introduction to the Mechanics of Continuous Media [in Russian], Fizmatgiz, Moscow (1962).
6. W. Prager, Introduction to the Mechanics of Continuous Media [Russian translation], IL (1963).
7. W. Wilkinson, Non-Newtonian Fluids [Russian translation], Mir (1964).
8. V. V. Novozhilov, Theory of Elasticity [in Russian], Sudpromgiz, Leningrad (1958).
9. M. Reiner, Rheology [Russian translation], Nauka (1965).
10. M. Kogan, Rarefied Gas Dynamics [in Russian], Nauka (1967).
11. L. Landau and E. Lifshits, Mechanics of Continuous Media [in Russian], GITTL (1954).
12. A. Einstein, Collected Works [Russian translation], Vol. 3, Nauka (1967).
13. G. Graber, S. Ark, and W. Grigull, Principles of the Science of Heat Exchange [Russian translation], IL (1958).
14. É. L. Aéro, A. N. Bulygin, and E. V. Kuvshinskii, Prikl. Mekh. Mat., 29, No. 2 (1965).
15. É. V. Kuvshinskii and É. L. Aéro, Fiz. Tverd. Tela, 5, No. 9 (1963).
16. É. L. Aéro and E. V. Kuvshinskii, Fiz. Tverd. Tela, 2, No. 7 (1960).
17. É. L. Aéro and E. V. Kuvshinskii, Fiz. Tverd. Tela, 6, No. 9 (1964).
18. R. A. Toupin, J. Rational Mech. and Analysis, 5, No. 6, 850 (1956).
19. Le Corre, Bull. Soc. Franc. Mineral et Cristallogr., 77, 1398 (1954).
20. R. Tiffen and A. Stevenson, Quart. J. Mech. Appl. Math., 19, 306 (1956).
21. E. S. Rajagopal, Ann. Phys., 6, Nos. 3-4, 192 (1960).
22. G. Grioli, Ann. Mat. Pura ed Appl., 50, 389 (1960).
23. R. D. Mindlin and H. F. Tiersten, Arch. Rational Mech. and Analysis, 11, No. 5, 415 (1962).
24. I. S. Dahler, J. Chem. Phys., 30, 1447 (1959).
25. I. S. Dahler and L. E. Scriven, Nature, 92, No. 4797 (1961).
26. H. Grad, Commun. Pure Appl. Math., 5, 455 (1952).
27. C. Truesdell, J. Rational Mech. and Analysis, 1, 158 (1952).
28. J. L. Ericksen and C. Truesdell, Arch. Rational Mech. and Analysis, 1, 295 (1958).
29. J. L. Ericksen, Arch. Rational Mech and Analysis, 4, 231 (1960).
30. J. L. Ericksen, Trans. Soc. Rheol., 4, 231 (1960).
31. N. Joel and W. A. Wooster, Acta Crist., 13, 516 (1960).
32. H. Jaffe and C. S. Smith, Phys. Rev., 121, 1604 (1961).